

MULTIDIMENSIONAL GRAVITY AND COSMOLOGY: EXACT SOLUTIONS

V.N. MELNIKOV

*Centre for Gravitation and Fundamental Metrology, VNIIMS,
3-1 M. Ulyanovoy Str., Moscow, 117313, Russia*

1 Introduction

It is well known that pioneering papers of T.K. Kaluza and O. Klein ^{1,2} on 5-dimensional gravity (see also ^{3,4,5,6}) opened the interest to investigations in multi-dimensional gravity. These ideas were continued by P.Jordan ⁷ who suggested to consider the more general case $g_{55} \neq \text{const}$ leading to the theory with an additional scalar field. The papers ^{1,2,7} were in some sense a source of inspiration for C.Brans and R.H.Dicke in their well-known work on the scalar-tensor gravitational theory ⁸.

In the 70th an interest to multidimensional gravitational models was stimulated mainly by: i) the ideas of gauge theories leading to the non-Abelian generalization of Kaluza-Klein approach ¹⁰ and by ii) supergravitational theories ^{9,10}. In the 80th the supergravitational theories were "replaced" by superstring models ¹¹. In all these theories 4-dimensional gravitational models with extra fields were obtained from some multidimensional model by dimensional reduction based on the decomposition of the manifold

$$M = M^4 \times M_{int},$$

where M^4 is a 4-dimensional manifold and M_{int} is some internal manifold (mostly considered as a compact one).

The earlier papers on multidimensional cosmology dealt with a block-diagonal cosmological metric defined on the manifold

$$M = R \times M_0 \times \dots \times M_n$$

of the form

$$g = -dt \otimes dt + \sum_{r=0}^n a_r^2(t) g^r$$

where (M_r, g^r) are Einstein spaces, $r = 0, \dots, n$ ¹³⁻³⁷.

In ^{23,24,28,33,37,34,41,52,53} the models with higher dimensional "perfect"-fluid were considered. In these models pressures (for any component) are proportional to the density

$$p_r = \left(1 - \frac{u_r}{d_r}\right) \rho,$$

$r = 0, \dots, n$, where d_r is a dimension of M_r . Such models are reduced to pseudo-Euclidean Toda-like systems with the Lagrangian

$$L = \frac{1}{2} G_{ij} \dot{x}^i \dot{x}^j - \sum_{k=1}^m A_k e^{u_k^i x^i}$$

and the zero-energy constraint $E = 0$. In a classical case exact solutions with Ricci-flat (M_r, g^r) for 1-component case were considered by many authors (see, for example, ^{21,22,33,34,52,53} and references therein). For the two component perfect-fluid there were solutions with two curvatures, i.e. $n = 2$, when $(d_1, d_2) = (2, 8), (3, 6), (5, 5)$ ⁸⁸ and corresponding non-singular solutions from ⁹². Among the solutions ⁸⁸ there exists a special class of Milne-type solutions. Recently some interesting extension of 2-component solutions were obtained in ⁸⁹.

It should be noted that the pseudo-Euclidean Toda-like systems are not well-studied yet. There exists a special class of eqs. of state that gives rise to the Euclidean Toda models. First such solution was considered in ⁵³ for the Lie algebra a_2 . Recently the case of $a_n = sl(n+1)$ Lie algebras was considered and the solutions were expressed in terms of a new elegant representation obtained by Anderson ⁸⁷.

The cosmological solutions may have regimes with: i) spontaneous and dynamical compactifications; ii) Kasner-like behavior near the singularity; iii) isotropization for large times (see, for example, ^{86,52}).

Near the singularity one can have an oscillating behavior like in the well-known mixmaster (Bianchi-IX) model. Multidimensional generalizations of this model were considered by many authors (see, for example, ^{56,57,58,59}). In ^{60,61} the billiard representation for multidimensional cosmological models near the singularity was considered and the criterion for the volume of the billiard to be finite was established in terms of illumination of the unit sphere by point-like sources. For perfect-fluid this was considered in detail in ⁶¹. Some interesting topics related to general (non-cosmological) situation were considered in ⁶².

Multidimensional cosmological models have a generalization to the case when the viscosity of the "fluid" is taken into account ⁹⁰. Some classes of exact solutions were obtained, in particular nonsingular cosmological solutions.

Multidimensional quantum cosmology based on the Wheeler-DeWitt (WDW) equation

$$\hat{H}\Psi = 0,$$

where Ψ is the so-called "wave function of the universe", was treated first in ³⁵, (see also ⁴⁷). This equation was considered for the vacuum case in ³⁵ and integrated in a very special situation of 2-spaces. The WDW equation for the "perfect-fluid" was investigated in ⁵². The exact solutions in 1-component case were considered very carefully in ⁷⁹ (for perfect fluid). In ⁴⁴ the multidimensional quantum wormholes were suggested, i.e. solutions with a special-type behavior of the wave function (see ⁴⁸).

These solutions were generalized to the perfect-fluid case in ^{52,79}. In ⁶¹ the "quantum billiard" was obtained for WDW solutions near the singularity. It should be also noted that the "third-quantized" multidimensional cosmological models were considered in several papers ^{43,84,79}.

Cosmological solutions are closely related to solutions with the spherical symmetry. The first multidimensional generalization of such type was considered by D. Kramer ⁶⁷ and rediscovered by A.I. Legkii ⁶⁸, D.J.Gross and M.J.Perry ⁶⁹ (and also by Davidson and Owen). In ⁷¹ the Schwarzschild solution was generalized to the case of n internal Ricci-flat spaces and it was shown that black hole configuration takes place when scale factors of internal spaces are constants. In ⁷² an analogous generalization of the Tangherlini solution ⁷⁰ was obtained. These solutions were also generalized to the electrovacuum case ^{73,76,74}. In ^{75,74} multidimensional dilatonic black holes were singled out. An interesting theorem was proved in ⁷⁴ that "cuts" all non-black-hole configurations as non-stable. In ⁷⁷ the extremely-charged dilatonic black holes were generalized to Majumdar-Papapetrou case when the cosmological constant is non-zero.

At present there exists a special interest to the so-called M-,F-theories etc. ^{95,97}. These theories are "supermembrane" analogues of superstring models in $D = 11, 12$ etc. ¹¹. The low-energy limit of these theories leads to models governed by the action

$$S = \int d^D x \sqrt{|g|} [\{R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a \in \Delta} \frac{\theta_a}{n_a!} \exp[2\lambda_a(\varphi)] (F^a)_g^2\}, \quad (1)$$

where g is metric, $F^a = dA^a$ are forms of rank $F^a = n_a$, and φ^α are scalar fields.

In ¹¹⁸ it was shown that after dimensional reduction on the manifold $M_0 \times M_1 \times \dots \times M_n$ and when the composite p -brane ansatz is considered the problem is reduced to the gravitating self-interacting σ -model with certain constraints imposed. For electric p -branes see also ^{107,108,119}. This representation may be considered as a powerful tool for obtaining different solutions with intersecting p -branes (analogs of membranes). In ^{118,119} the Majumdar-Papapetrou type solutions were obtained (for non-composite case see ^{107,108}). These solutions correspond to Ricci-flat (M_i, g^i) , $i = 1, \dots, n$, and were generalized also to the case of Einstein internal spaces. Earlier some special classes of these solutions were considered in ^{102,103,111,112,113}. The obtained solutions take place, when certain orthogonality relations (on couplings parameters, dimensions of "branes", total dimension) are imposed. In this situation a class of cosmological and spherically-symmetric solutions was also obtained ¹²³. Special cases were also considered in ^{105,122,121}. The solutions with the horizon were also considered in details in ^{106,114,116}. In ¹²⁵ some propositions related to i) interconnection between the Hawking temperature and the singularity behaviour, and ii) to multitemporal configurations were proved.

It should be noted that the multidimensional and multitemporal generalization of the Schwarzschild and Tangherlini solutions were considered in ^{76,78}, where the generalization of Newton's formulas on multitemporal case was obtained.

We note also that there exists a large variety of Toda solutions (open or closed) when certain intersection rules are satisfied ¹²³.

In ¹²³ the Wheeler-DeWitt equation was also integrated for intersecting p -branes (in orthogonal case). The slightly-different approach was considered also in ¹²⁶. These solutions may be considered as an interesting first step for quantum description of low-dimensional supergravity theories in different super- p -branes theories.

Problems of multidimensional gravitational theories and models were discussed

at a variety of conferences during last years such as Marcel Grossmann meetings, conferences in Russia in Jaroslavl (1994), Novgorod (1996) and Ulyanovsk (1997). The whole session was devoted to these topics at the 8 Marcel Grossmann meeting in Jerusalem (June 1997).

2 Review of reports

Here are some short review of main talks submitted and discussed at this section.

The talk of E.I.Guendelman and A.B.Kaganovich was devoted to "Gravitational Theory without the Cosmological Constant Problem". It is known that the vacuum energy density or cosmological constant is predicted by modern particle theory to be very large ¹²⁸. Such parameter controls the large scale structure of the universe, which is consistent with a zero cosmological constant. A new approach for the solution of the cosmological constant problem developing a new gravitational theory ^{129 - 131} is offered where the measure of integration in the action principle is not necessarily $\sqrt{-g}$ but it is determined dynamically through additional degrees of freedom. This theory is based on the demand that such measure respect the principle of "non gravitating vacuum energy" ¹²⁹ which states that the Lagrangian density L can be changed to $L + \text{constant}$ without affecting the dynamics. Formulating the theory using the metric and the affine connection, as well as the fields which define the measure of integration, as independent dynamical variables they get as a consequence of the variational principle a constraint that enforces the vanishing of the cosmological constant ^{130, 131}. A successful model ¹³¹ that implements these ideas is realized in a six or higher dimensional space-time. The compactification of extra dimensions into a sphere gives the possibility of generating scalar masses and potentials, gauge fields and fermionic masses. It turns out that remaining four dimensional space-time must have effective zero cosmological constant. Another model ¹³², based on the same principles, but formulated in four-dimensions, can incorporate a period of inflation for the early universe before making a transition to a zero cosmological constant phase which is realized without fine tuning.

The talk of V. D. Ivashchuk and V. N. Melnikov, "Exact Solutions in Multi-dimensional Gravity with Intersecting p-branes" was devoted to multidimensional gravitational models containing several dilatonic scalar fields and antisymmetric forms (see (1)). The manifold is chosen in the form $M = M_0 \times M_1 \times \dots \times M_n$, where M_i are Einstein spaces ($i \geq 1$). The block-diagonal metric is chosen and all fields and scale factors of the metric are functions on M_0 . For the forms composite (electro-magnetic) p-brane ansatz is adopted. The model is reduced to gravitating self-interacting sigma-model with certain constraints. In pure electric and magnetic cases the number of these constraints is $n_1(n_1 - 1)/2$ where n_1 is the number of 1-dimensional manifolds among M_i . In the "electro-magnetic" case for $\dim M_0 = 1, 3$ additional n_1 constraints appear. A family of "Majumdar-Papapetrou type" solutions governed by a set of harmonic functions is obtained, when all factor-spaces M_ν are Ricci-flat. These solutions are generalized to the case of non-Ricci-flat M_0 when also some additional "internal" Einstein spaces of non-zero curvature are added to M . As an example exact solutions for $D = 11$ supergravity and related 12-dimensional theory ¹¹⁷ are presented.

In a paper of S.S.Kokarev "New point of view on space-time dynamics" it was suggested, that a curved 4-dimensional space-time manifold is a surface of a strained elastic plate in multidimensional embedding space-time. All thicknesses h_m of the plate along extra dimensions are much less than its 4-dimensional sizes. Multidimensional elastic free energy density integrated over extra (internal) coordinates of the strained plate has the form

$$F_{\text{pl}} = \frac{\mu H_N h_m^2}{12} \eta^{mn} \int \sqrt{-g} d^4 x \{ \xi_{m,\mu,\nu} \xi_n^{\mu,\nu} + f \xi_{m,\mu} \xi_{n,\nu} \}. \quad (2)$$

where μ, f are "phenomenological" elastic constants of the theory, H_N is the product of all thicknesses of the plate, ξ^m are components of a displacement vector, η_{mn} is the part of a metric of the embedding space, $g_{\mu\nu}$ is a metric on the plate surface. This expression has been derived for a weak straining case, and is based on the multidimensional Hook's law. It is similar to a Lagrangian density of a gravitational field in GR up to a multiplicative constant, if the thicknesses in extra dimensions are equal to each other, the Poisson coefficient of a material of the space-time plate σ is equal to $1/2$ ($f = -1$). Dimensional analysis gives for the Yung modulus of a space-time substance value $\sim 10^{10^2}$ Pa. Variation of the (2) over ξ^m leads to bewave equation for displacement vector components

$$D_m \square^2 \xi^m = P^m, \quad (\text{no summation!})$$

and to the boundary conditions at the boundary of the plate:

$$D_m \oint_{\partial\Sigma} d^3 S^\mu \square \xi_{m,\mu} \delta \xi^m + \frac{D_m}{f+1} \oint_{\partial\Sigma} d^3 S^\mu (\xi_{m,\mu,\lambda} + \eta_{\mu\lambda} f \square \xi_m) \delta \xi^{\mu\lambda} = 0.$$

Here D_m is the cylindrical stiffness factor of the 4-dimensional plate in m -th extra dimension ($m = \overline{1, N}$), depending on elastic constants of the theory, N is the number of additional four dimensions of the embedding space, P_m is normal to V_4 component of an external multidimensional forces, bending the plate.

GR in such approach corresponds to degenerate case, since under $\sigma = 1/2$ (and only in this case!) free energy density is a total divergence. Basic notions and relations of GR acquire in this approach a new interpretation. For example, Einstein equations appear as some special relation between normal and tangent stresses of the space-time plate.

The talk by U.Günther and A.Zhuk "Gravitational Excitons From Extra Dimensions" was devoted to stability conditions for compactified internal spaces. Starting from a multidimensional cosmological model after a dimensional reduction an effective four-dimensional theory in Brans - Dicke and Einstein frames was obtained. The Einstein frame was considered as the physical one. In this frame they derived an effective potential. It was shown that small excitations of the scale factors $a_i = \exp(\beta^i)$, ($i = 1, \dots, n$) of internal spaces near minima of the effective potential have a form of massive scalar particles (gravitational excitons) developing in the external space - time. The exciton masses strongly depend on the dimensions and curvatures of the internal spaces, and possibly present additional fields living on the internal spaces. These fields will contribute to the effective potential, e.g.

due to the Casimir effect, and by this way affect the dynamics of the scale factor excitations. So, the detection of the scale factor excitations can not only prove the existence of extra dimensions, but also give additional information about the dimension of the internal spaces and about fields possibly living on them.

For some particular classes of effective potentials with one, two and n scale factors exciton masses as functions of parameters of the internal spaces were calculated and stability criterions necessary for the compactification of the spaces were derived.

Their analysis shows that conditions for the existence of stable configurations may depend not only on dimension and topology of the internal spaces, and additional fields contributing to the effective potential, but also on the number of independently oscillating scale factors. For example, n -scale-factor models with a saddle point as extremum of the effective potential $U_{eff}(\beta_1, \dots, \beta_n)$ would lead to an unstable configuration. Masses of the corresponding excitations would be positive (excitons) as well as negative (tachyons). Under scale factor reduction to an m -scale factor model with $m < n$, i.e. when some of the scale factors were connected by constraints $\beta_i = \beta_k$, the saddle point may, for certain potentials, reduce to a stable minimum point of the new effective potential $U_{eff}(\beta_1, \dots, \beta_m)$. As a result all masses of excitations would be positive (excitons). This "stabilization via scale factor reduction" is demonstrated explicitly on a model with one-component perfect fluid.

In Paulo Gali Macedo talk "Anti-gravity Effect in Jordan-Thiry Unified Field Theory" it was pointed out that Jordan-Thiry theory tries to unify gravity and electromagnetism in a 5 dimensional space-time. However, in order to reach that goal, to be self-consistent and to be fully covariant such a theory cannot contain (when reduced to 4 dimensions) only these two fields but needs the presence of a third field which is a scalar one.

This field is coupled to the other two fields through the field equations which arise in a 4 dimensional space-time from the dimensional reduction of the 5 dimensional Einstein equations. In particular, these equations describe the induction of the gradient of such scalar field by an electromagnetic field in much the same way as in electromagnetism a magnetic field can arise from the time variation of an electric field.

On the other hand, the equation of motion of particles subject to the unified field, which is the 5 dimensional geodesic equation, when dimensionally reduced to 4 dimensions contains force terms due to each of the 3 fields individually. Assuming that a neutral extended body has a non zero scalar charge if it is made of charged particles at the microscopic level, this was shown to generate an anti-gravity effect in neutral bodies if the term due to the scalar field cancels the gravitational term in the equation of motion.

The author also presented a new solution for the field equations corresponding to the propagation of coupled scalar-electromagnetic waves with a dispersion relation showing possible sub and supraluminal propagation depending on the unperturbed electromagnetic fields present in the region.

The author conjectured that this effect might be a possible explanation for the Nodlan-Ralston effect.

The paper by Yu. S. Vladimirov and S. I. Mamontov "A 6-dimensional geometric model of gravi-electroweak interactions" states that in the framework of a 6-dimensional geometric model of physical interactions of Kaluza-Klein type it is possible to entirely unify general relativity with the Weinberg-Salam theory of electroweak interactions.

In this 6-dimensional model,

- (1) for additional coordinates the topology of a 2-torus is used,
- (2) the x^5 - and x^6 -dependence of the mixed components of the 6-dimensional metric is introduced and
- (3) the reduction to a 4-dimensional theory is carried out using the dyad method in a gauge like the 4-dimensional chronometric one.

It is shown that (a) the hypercharge Y and the isospin projection T_3 are eigenvalues of dyadic operators along the directions of two additional coordinates, (b) an arbitrariness in indentifying the mixed components of the metric tensor with the four physical vector fields is revealed and (c) an arbitrariness in the definition of spinor physical fields (the first generation lepton doublet) is established.

In the talk of V.D.Dzhunushaliev "Multidimensional $SU(2)$ wormhole bounded by two null surfaces" the multidimensional gravity on principal bundle with structural $SU(2)$ gauge group was considered¹⁴³. In this case the multidimensional metric has the following components: 4D Einstein's metric on the base of bundle, the diagonal metric on the fibre of bundle (fibre is a gauge group = symmetric space), nondiagonal components that equivalent (in Kaluza - Klein's sense) to gauge (electromagnetic or Yang - Mills) fields. The multidimensional metric has the form:

$$ds^2 = e^{2\nu(r)} dt^2 - r_0^2 e^{2\psi(r)} \sum_{a=1}^3 (\sigma^a - A_\mu^a(r) dx^\mu)^2 - dr^2 - a^2(r) (d\theta^2 + \sin^2 \theta d\varphi^2).$$

The "potentials" A_μ^a have the monopole-like form. The solution of corresponding Einstein's vacuum equation gives a multidimensional wormhole located between two surfaces ($r = \pm a_0$):

$$\begin{aligned} \nu &= -3\psi, \\ a^2 &= a_0^2 + r^2, \\ e^{-\frac{4}{3}\nu} &= \frac{q}{2a_0} \cos \left(\sqrt{\frac{8}{3}} \arctan \frac{r}{a_0} \right), \\ v &= \sqrt{6} \frac{a_0}{r_0 q} \tan \left(\sqrt{\frac{8}{3}} \arctan \frac{r}{a_0} \right). \end{aligned}$$

The metric on surfaces ($r = \pm a_0$) is not singular but $ds^2 = 0$. As the nondiagonal components are similar to a gauge fields (in Kaluza - Klein's sense) then in this sense this solution is dual to the black hole in 4D Einstein - Yang - Mills gravity: 4D black hole has the stationary area outside the event horizon but multidimensional wormhole inside the null surface (on which $ds^2 = 0$). Similar solution was received for standard 5D Kaluza - Klein's theory¹⁴⁴.

In the talk of Spiros Cotsakis "Mathematical Problems in Quadratic and Higher-Order Gravity and Cosmology" it was stated that although higher order gravity theories have been considered in the past as a means of a first approximation to quantum approaches to gravitation and also providing better singularity behavior (singularity avoidance), they have also been criticised on being unphysical, not always having a well-posed initial value formulation and generally leading to field equations of higher than second order, a feature that usually makes their analyses intractable. Several other ways of motivating further study of these theories, in particular use these theories as a testing ground for checking which *properties* (e.g., black hole entropy, inflation, isotropization, recollapse etc.) may prove to be fundamental, were discussed. Also the question was discussed of whether dynamical equivalence between the two conformally related spacetimes, which result from the conformal transformation that maps the space of higher order theories of gravity to that of general relativity plus scalar fields with non-trivial self interaction potentials, implies their physical equivalence. He showed that at least for manifolds of a warped product form with base a compact Riemannian manifold with negative Ricci curvature and fiber with radius the scalar field generated by the conformal transformation, the two theories cannot be physically equivalent. He presented detailed perturbation results which support the past instability conjecture namely, that all homogeneous and isotropic, physically reasonable cosmological solutions of general relativity are past unstable in the framework of higher order gravity theories. The issue of constrained (Palatini) variations for an arbitrary higher order lagrangian with general matter couplings and general symmetric connection was considered. He proved a theorem stating that the field equations are identical to those obtained from the usual Hilbert variation. This has the interesting consequence that it directly provides a generalization of the conformal equivalence theorem for arbitrary Weyl geometries. It was shown that the quasi-exponential solution of quadratic gravity is an attractor of all homogeneous and isotropic solutions of the general $f(R)$ theory. In quadratic theories, de Sitter space is an attractor of the Bianchi IX universe provided the scalar 3-curvature does not exceed the value of the general scalar field potential associated with the conformal transformation. This result settles the cosmic no-hair conjecture in the general case without assuming particular forms (for instance exponential) for the self-interacting potential. The proof relies on rigorous estimates of the possible bounds of a certain function (called the Moss-Sahni function) and a nontrivial argument that connects the behavior of that function to the spatial part of the scalar curvature. A generalization of the Collins-Hawking theorem for higher order gravity theories was proved. The set of homogeneous and anisotropic cosmologies that can approach isotropy at late times is of measure zero in the space of all spatially homogeneous universe models. He also provided a list of open problems in the field.

The paper of A.M. Baranov and N.M. Bardushko "Algebraic classification of Kaluza space with stationary electromagnetic and scalar fields alone" deals with an algebraic classification of electromagnetic scalar fields without sources and gravitational field potentials in the five-dimensional Kaluza space¹⁵². A general problem of the algebraic classification of the Kaluza space is investigated in¹⁵³.

Five-dimensional metric components are given as $G_{ij} = \delta_{ij} = \text{diag}(1, -1, -1, -1);$

$G_{j5} = A_j$; $G_{55} = -1$ $\mu\nu = 0, 1, 2, 3, 5$; $i, j = 0, 1, 2, 3$; A_j are the electromagnetic 4-potentials. A cylindrical condition with respect to 5-th coordinate is valid. The Weyl 4-tensor is mapped on a 10-dimensional bivector real space with metric $G_{AB} = \text{diag}(1, 1, 1, -1, -1, -1; 1, -1, -1, -1)$ ($A, B = 1, 2, \dots, 10$). Then the traceless symmetric 10×10 real Weyl matrix has the block form $W = \begin{pmatrix} 0 & F \\ F & 0 \end{pmatrix}$, where a rectangular 4×6 matrix F has the block structure $F = \begin{pmatrix} E & e \\ B & b \end{pmatrix}$ with traceless symmetric 3×3 matrices $E = (E_{i,j})$; $B = (B_{i,j})$ and vector-columns $e = (E_{i,0})$; $b = (B_{i,0})$. Here E_i is an electric field strength and B_i is a magnetic displacement vector.

The algebraic classification may be made most simply in the stationary case when $e = b = 0$. Then λ -matrix $W(\lambda)$ is described as

$$W(\lambda) = \begin{pmatrix} -\lambda I & 0 & E & 0 \\ 0 & -\lambda I & -B & 0 \\ E & -B & -\lambda I & 0 \\ 0 & 0 & 0 & -\lambda \end{pmatrix}, \quad (3)$$

where I is an identity 3×3 matrix. Under elementary transformations matrix $W(\lambda)$ is reduced to the block diagonal form $W(\lambda) = \text{diag}(I; \lambda I; \lambda I(E^2 + B^2 - \lambda^2 I); \lambda)$.

A minimal annihilating polynomial of such matrix is $p(\lambda) = \lambda(\lambda^6 + a_5\lambda^5 + \dots + a_1\lambda + a_0)$ and a "total" annihilating polynomial of the λ -matrix $W(\lambda)$ may be written as $P(\lambda) = \lambda^4 p(\lambda)$. Finally the normal canonical form of λ -matrix $C(\lambda) = \text{diag}(I, I, \lambda, \lambda I(E^2 + B^2 - \lambda^2 I))$ is obtained.

An eigenvalue problem is reduced to the solution of the six power algebraic equation for λ or a cubic equation for $\Lambda = \lambda^2$: $\det(E^2 + B^2 - \Lambda I) = 0$. The reality of Weyl matrix gives an existence of four analogs of Petrov algebraic types here: $I, I_a, D, 0$, where the matrix of I_a type has eigenvalues for example: $\lambda_1 = \dots = \lambda_8 = 0$; $\lambda_9 = -\lambda_{10} = \lambda$.

When there is an electric field strength E_j alone (or B_j) then

$$W(\lambda) = \text{diag}(Q(\lambda), \bar{Q}(\lambda), \lambda I, \lambda),$$

where $Q(\lambda) = \lambda I + iE$ (or $\lambda I + iB$), $i^2 = -1$, the complex conjugate is denoted by the bar. In particular the electric field of a charged pencil has the I_a algebraic type, the Coulomb field has D type, and the charged plane has 0 type.

In the case of scalar field $g_{55} = \varphi$ without electromagnetic and gravitational fields for locally geodesic coordinates there exists the traceless symmetric 10×10 real Weyl matrix with a matrix block $W = \begin{pmatrix} 0 & 0 \\ 0 & \Phi \end{pmatrix}$, where the 4×4 matrix Φ corresponds to the scalar field, $\Phi = (\varphi_{i,j})$.

A problem of the algebraic classification is reduced to an eigenvalue problem of the symmetric real traceless Weyl matrix with the characteristic equation $\det(\lambda I) \cdot \det(\lambda I) \cdot \det(\Phi - \lambda \Delta) = 0$; I is an identity 3×3 matrix. $\Delta = \text{diag}(1, -1, -1, -1)$. The traceless of Weyl matrix is an equivalent to a wave equation $\square\varphi = 0$, i.e. we have a massless scalar field. For example, when $\varphi = \varphi(u)$, $u = x^0 - x^1$ (a plane

wave) we have $[(1,1,2)]$ algebraic type, which is also the type of an asymptotic field of spherical and cylindrical scalar waves.

In a stationary case $\Phi = \text{diag}(0, S)$; $S = (\varphi_{,a,b})$; $\Delta\varphi = 0$; $a, b = 1, 2, 3$. For a Coulomb-like static scalar field we have the algebraic type $[(1)(1,1)(1)]$, i.e. D type.

The paper of A.M. Baranov "Kaluza space and magnetic charge" considers a bivector $F_{\mu\nu}$ as a generalization of the Maxwell tensor of electromagnetic field in Kaluza five dimensional space-time (with time-like direction alone) with $F_{5i} = -A_{5,i} = C_i$; $C_0 = A_{5,0} = 0$ ($i, j = 0, 1, 2, 3$) and the cylindrical condition with respect to x^5 coordinate. A dual rotationed the Kaluza space-time is defined: \star (see ¹⁵²) as $\star F_{\mu\nu} = (1/2)\varepsilon_{\rho\mu\nu\alpha\beta} F^{\alpha\beta} u^\rho$, where $u^\rho = \delta_0^\rho$. Then 3-vector \vec{C} may be connected with a magnetic displacement 3-vector in usual space as $\vec{C} = \star \vec{B}$. The standard dual rotation is defined by $\star F_{\mu\nu} = (1/2)\varepsilon_{\rho\mu\nu\alpha\beta} F^{\alpha\beta} \lambda^\rho$, where $\lambda^\rho = \delta_5^\rho$. Using repeatedly these operations we have $\star^2 = +1$; $\star^2 = -1$.

For stationary magnetic fields without current sources we have $\text{rot}\vec{B} = 0$. The solution of this equation is gradient, $\vec{B} = -\nabla\varphi$, where φ is a magnetic potential. Such approach does not correspond to the well-known definition of electromagnetic field tensor by means of 4-potential. Defining by this way magnetic field may be connected with the gradient vector field \vec{C} , which is the part of 5-dimensional bivector $F_{\mu\nu}$, by the operation $\star : \star C_a = -\star (\nabla A_5)_a = (1/2) u^\rho \varepsilon_{\rho 5ab0} F_{b0} = B_a$ and conversely, $\star B_a = C_a = -(\nabla A_5)_a$; $a, b = 1, 2, 3$. Thus in 4-dimensional space-time \vec{B} and \vec{C} are different vectors: one has the rotational nature and the other has the gradient nature ($A_5 = \varphi$ is a magnetic potential).

The first two equations are the usual Maxwell equations and the last equation is connected with the 5th component of the 5-current density vector which is a source of a magnetic potential: $\vec{C} = -\nabla\psi \equiv -\nabla A_5$ and $\Delta A_5 = \Delta\psi = -4\pi j^5$.

A continuity equation for 5-current in 5-dimensional space-time is $j^\mu{}_{,\mu} = j^0{}_{,0} + \text{div}\vec{j} = 0$, where the cylindrical condition is used. The equation is reduced by the dual rotations $\star \star$ into $j^5{}_{,0} = 0$, i.e. j^5 does not depend on time. In this case one has ¹³⁸

$$\star \star j^0 = j^5 \quad \text{and} \quad \star \star j^5 = -j^0, \quad (4)$$

where $\star \star \vec{j} = \star \star \vec{j} = 0$.

In other word, in the 5-dimensional space-time one may connect electric and magnetic charges by the dual rotations \star and \star . It should be noted that the magnetic charges are not observed in 4D space-time. When the point charges are considered the components j^0 and j^5 may be written as $j^0 = \mathbf{e}\delta(\vec{r})$ and $j^5 = \mathbf{m}\delta(\vec{r})$, where \mathbf{e} is an electric charge and \mathbf{m} is a magnetic charge; $\delta(\vec{r})$ is the 3-space δ -function. Therefore one may write the connection between these charges ¹³⁸

$$\star \star \mathbf{e} = \mathbf{m} \quad \text{and} \quad \star \star \mathbf{m} = -\mathbf{e}. \quad (5)$$

Thus, this approach admits an existence of "double-dual" pairs of static magnetic and electric charges.

In the paper of F. Burgbacher, C. Lämmerzahl and A. Macias "Reasons for the space-time to be four-dimensional" it was shown that (contrary to

statements found in the literature) stable atoms may exist in higher-dimensional space-time.

In showing this, only very few fundamental quantum principles are used. Two consequences are drawn from this result. First, it is possible to determine the dimensionality of space-time from the structure of the spectra of the hydrogen atoms, and second that the Maxwell equations in higher dimensions are in general non-local and do not obey Gauss law.

The equation describing a hydrogen atom is the Schrödinger equation.

This can be based on two reasons:

- (i) The form of kinetic energy is independent of the dimension.
- (ii) If one makes an axiomatic approach to quantum theory ¹⁵⁷, then one arrives at a modified Dirac equation, the non-relativistic limit of which gives a Schrödinger equation with the d -dimensional Laplacian as kinetic term.

Consequently, one takes as general ansatz for the Hamilton operator for the hydrogen atom in d spatial dimensions $H = -\frac{\hbar^2}{2m}\Delta_d + V(r)$ where $V(r)$ is a spherically symmetric potential. One can show that (i) a potential $\sim 1/r$ (that is $\kappa = 1$) of a point charge in higher dimensions leads to stable atoms in higher dimensions, (ii) the dimensionality enters the atomic spectra thus making it possible to infer from spectroscopy the three-dimensionality of space, and (iii) the Maxwell equations have to be modified in order to allow solutions of the form $1/r$. (Other reasons for the four-dimensionality of space-time based on the propagation of helicity states and of Huygen's principle have been given in ¹⁵⁶.)

The structure of the Maxwell equations change in higher dimensions. The equation for the electric potential ϕ can be given in terms of Riesz distributions

$$\left(\overline{\Delta_d^{\frac{d-1}{2}}} * \phi\right)(x) = (4\pi)^{\frac{d-1}{2}} \Gamma\left(\frac{d-1}{2}\right) \rho(x)$$

where $\rho(x)$ is the charge density. $\overline{\Delta_d^{\frac{d-1}{2}}}$ is the operator which replaces the Laplacian in three dimensions. In general this operator is not a differential operator. This operator has the feature that Gauss law is no longer valid.

In the talk of Thomas Klösch and Thomas Strobl "Complete Classification of 1+1 Gravity Solutions" a classification of the maximally extended solutions for all 1+1 gravity models (comprising e.g. generalized dilaton gravity as well as models with non-trivial torsion) was presented. No restrictions are placed on the topology of the arising solutions, and indeed it was found that for generic models solutions on non-compact surfaces of arbitrary genus with an arbitrary non-zero number of holes can be obtained. The classical solution spaces (solutions of the field equations with fixed topology modulo gauge transformations) are parametrized explicitly and a geometrical interpretation of the resulting parameters is provided. This allows also to address various issues in a Hamiltonian treatment and in canonical quantization of gravity.

References

1. T. Kaluza, *Sitzungsber. Preuss. Akad. Wiss. Berlin Phys. Math.*, **K1** 33, 966 (1921).
2. O. Klein, *Z. Phys.* **37**, 895 (1926).

3. V. De Sabbata and E. Schmutzer, *Unified Field Theories in more than Four Dimensions*, (Scientific World, Singapore, 1982).
4. H. C. Lee, *An Introduction to Kaluza-Klein Theories*, (World Scientific, Singapore, 1984).
5. Yu.S. Vladimirov *Physical Space-Time Dimension and Unification of Interactions* (University Press, Moscow, 1987) (in Russian).
6. P.S. Wesson and J. Ponce de Leon, *Gen. Rel. Gravit.* **26**, 555 (1994).
7. P.Jordan, "Erweiterung der projektiven Relativitätstheorie", *Ann. der Phys.* 219 (1947).
8. C. Brans and R.H. Dicke, *Phys. Rev. D* **124**, 925 (1961).
9. E. Cremmer, B. Julia, and J. Scherk, *Phys. Lett.* **B76** 409 (1978).
10. A. Salam and E. Sezgin, eds., *Supergravities in Diverse Dimensions*, reprints in 2 vols., (World Scientific, Singapore, 1989).
11. M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory* (Cambridge University Press., Cambridge, 1987).
12. V.A. Belinskii and I.M. Khalatnikov, *ZhETF*, **63**, 1121 (1972).
13. P. Forgacs and Z. Horvath, *Gen. Rel. Grav.* **11**, 205 (1979).
14. A. Chodos and S. Detweyler, *Phys. Rev. D* **21**, 2167 (1980).
15. P.G.O. Freund, *Nucl. Phys.* **B 209**, 146 (1982).
16. R. Abbot, S. Barr and S. Ellis, *Phys. Rev. D* **30**, 720 (1984).
17. V.A. Rubakov and M.E. Shaposhnikov, *Phys. Lett.* **B 125** 136 (1983).
18. D. Sahdev, *Phys. Lett.* **B 137**, 155 (1984).
19. E. Kolb, D. Linkley and D. Seckel, *Phys. Rev. D* **30** 1205 (1984).
20. S. Ranjbar-Daemi, A. Salam and J. Strathdee, *Phys. Lett.* **B 135**, 388 (1984).
21. D. Lorentz-Petzold, *Phys. Lett.* **B 148** 43 (1984).
22. R. Bergamini and C.A. Orzalesi, *Phys. Lett.* **B 135**, 38 (1984).
23. M. Gleiser, S. Rajpoot and J.G. Taylor, *Ann. Phys. (NY)* **160**, 299 (1985).
24. U. Bleyer and D.-E. Liebscher, in *Proc. III Sem. Quantum Gravity* ed. M.A.Markov, V.A.Berezin and V.P. Frolov (Singapore, World Scientific, 1985) p. 662.
25. U. Bleyer and D.-E. Liebscher, *Gen. Rel. Gravit.* **17**, 989 (1985).
26. M. Demianski, Z. Golda, M. Heller and M. Szydlowski, *Class. Quantum Grav.* **3**, 1190 (1986).
27. D.L. Wiltshire, *Phys. Rev. D* **36**, 1634 (1987).
28. U. Bleyer and D.-E. Liebscher, *Annalen d. Physik (Lpz)* **44** 81 (1987).
29. G.W. Gibbons and K. Maeda, *Nucl. Phys. B* **298**, 741 (1988).
30. Y.-S. Wu and Z. Wang, *Phys. Rev. Lett.* **57** 1978 (1986).
31. G.W. Gibbons and D.L. Wiltshire, *Nucl. Phys. B* **287**, 717 (1987).
32. V.D. Ivashchuk and V.N.Melnikov, *Nuovo Cimento* **B 102**, 131 (1988).
33. K.A. Bronnikov, V.D. Ivashchuk and V.N. Melnikov, *Nuovo Cimento* **B102**, 209 (1988).
34. V.D. Ivashchuk and V.N. Melnikov, *Phys. Lett.* **A 135**, 465 (1989).
35. V.D. Ivashchuk, V.N. Melnikov and A.I. Zhuk, *Nuovo Cimento* **B 104**, 575 (1989).
36. V.A. Berezin, G. Domenech, M.L. Levinas, C.O. Lousto and N.D. Umerez, *Gen. Relativ. Grav.* **21**, 1177 (1989).

37. V.N. Melnikov in *Cosmology and Gravitation*, ed. M. Novello (Frontieres, Singapore, 1994) p. 147.
38. V.D. Ivashchuk and V.N. Melnikov, *Chinese Phys. Lett.* **7**, 97 (1990).
39. M. Demiansky and A. Polnarev, *Phys. Rev. D* **41**, 3003 (1990).
40. S.B. Fadeev, V.D. Ivashchuk and V.N. Melnikov in *Gravitation and Modern Cosmology* (Plenum, N.-Y., 1991) p. 37.
41. U. Bleyer, D.-E. Liebscher and A.G. Polnarev, *Class. Quant. Grav.* **8**, 477 (1991).
42. V.D. Ivashchuk, *Phys. Lett. A* **170**, 16 (1992).
43. A. Zhuk, *Class. Quant. Grav.* **9**, 202 (1992).
44. A. Zhuk, *Phys. Rev. D* **45**, 1192 (1992).
45. A. Zhuk, *Sov. Journ. Nucl. Phys.* **55**, 149 (1992).
46. A.I. Zhuk, *Sov. Journ. Nucl. Phys.* **56**, 223 (1993).
47. J.J. Halliwell, *Phys. Rev. D* **38**, 2468 (1988).
48. S.W. Hawking and D.N. Page, *Phys. Rev. D* **42**, 2655 (1990).
49. H. Liu, P.S. Wesson and J. Ponce de Leon, *J. Math. Phys.* **34** (1993), 4070.
50. V.R. Gavrilov, *Hadronic J.* **16** (1993), 469.
51. V.D. Ivashchuk and V.N. Melnikov, *Teor. Mat. Fiz.* **98** (1994), 312 (in Russian).
52. V.D. Ivashchuk and V.N. Melnikov, "Multidimensional cosmology with m -component perfect fluid" gr-qc/ 9403063; *Int. J. Mod. Phys. D* **3** (1994), 795.
53. V.R. Gavrilov, V.D. Ivashchuk and V.N. Melnikov, "Integrable pseudo-euclidean Toda-like systems in multidimensional cosmology with multicomponent perfect fluid", *J. Math. Phys.* **36**, 5829 (1995).
54. U. Bleyer and A. Zhuk, "Multidimensional integrable cosmological models with negative external curvature", *Gravitation and Cosmology*, **2** 106 (1995).
55. A.I. Zhuk, *Sov. Journ. Nucl. Phys.* **58**, No 11 (1995).
56. V.A. Belinskii and I.M. Khalatnikov, *ZhETF*, **63**, 1121 (1972).
57. J.D. Barrow and J. Stein-Schabes, *Phys. Rev. D* **32**, 1595 (1985).
58. J. Demaret, M. Henneaux and P. Spindel, *Phys. Lett. B* **164** 27 (1985).
59. M. Szydlowski and G. Pajdosz, *Class. Quant. Grav.* **6** (1989), 1391.
60. V.D. Ivashchuk, A.A. Kirillov and V.N. Melnikov, *Pis'ma ZhETF* **60** No 4 (1994), 225; *Izv. Vuzov (Fizika)* No 11 (1994) (in Russian).
61. V.D. Ivashchuk and V.N. Melnikov, "Billiard representation for multidimensional cosmology with multicomponent perfect fluid near the singularity", *Class. Quant. Grav.* **12**, 809 (1995).
62. A.A. Kirillov and V.N. Melnikov, "On Properties of Metrics Inhomogeneities in the Vicinity of a Singularity in K-K Cosmological Models", *Astron. Astrophys. Trans.*, **10**, 101 (1996).
63. M. Rainer, *Gravitation and Cosmology* **1**, 81 (1995).
64. M. Gasperini and G. Veneziano, *Phys. Rev. D* **50** (1994), 2519.
65. M. Gasperini and G. Veneziano, *Mod. Phys. Lett. A* **8** (1993), 701.
66. C. Angelantonj, L. Amendola, M. Litterio and F. Occhionero, "String cosmology and inflation", *Phys. Rev. D* **51**, 1607 (1995).
67. D. Kramer, *Acta Physica Polonica* **2**, F. 6, 807 (1969).

68. A.I. Legkii, in *Probl. of Grav. Theory and Elem. Particles* (Atomizdat, Moscow) **10**, 149 (1979) (in Russian).
69. D.J. Gross and M.J. Perry, *Nucl. Phys.* **B 226**, 29 (1993).
70. F.R. Tangherlini, *Nuovo Cimento* **27**, 636 (1963).
71. K.A. Bronnikov, V.D. Ivashchuk in *Abstr. VIII Soviet Grav. Conf* (Erevan, EGU, 1988) p. 156.
72. S.B. Fadeev, V.D. Ivashchuk and V.N. Melnikov, *Phys. Lett.* **A 161**, 98 (1991).
73. S.B. Fadeev, V.D. Ivashchuk and V.N. Melnikov, *Chinese Phys. Lett.* **8**, 439 (1991).
74. K.A. Bronnikov and V.N. Melnikov, *Annals of Physics (N.Y.)* **239**, 40 (1995).
75. U. Bleyer and V.D. Ivashchuk, *Phys. Lett.* **B 332**, 292 (1994).
76. V.D. Ivashchuk and V.N. Melnikov, *Class. Quantum Grav.*, **11**, 1793 (1994).
77. V.D. Ivashchuk and V.N. Melnikov, "Extremal Dilatonic Black Holes in String-like Model with Cosmological Term", *Phys. Lett.* **B 384**, 58 (1996).
78. V.D. Ivashchuk and V.N. Melnikov, *Int. J. Mod. Phys.* **D 3**, No 4, 795 (1994).
79. V.D. Ivashchuk and V.N. Melnikov, *Gravitation and Cosmology* **1**, No 3, 204 (1995).
80. V.A. Rubakov, *Phys. Lett.* , **B 214**, 503 (1988).
81. S. Giddings and A. Strominger, *Nucl. Phys.* **B 321**, 481 (1989).
82. A.A. Kirillov, *Pis'ma ZhETF* **55**, 541 (1992).
83. E.I. Guendelman and A.B. Kaganovich, *Phys. Lett.* **B 301**, 15 (1993).
84. T. Horigushi, *Mod. Phys. Lett.* **A 8**, 777 (1993).
85. V.N. Melnikov, "Multidimensional Cosmology and Gravitation", CBPF-MO-002/95, Rio de Janeiro, 1995, p. 210.
V.N. Melnikov in *Gravitation and Cosmology* ed. M. Novello (Frontieres, Singapore, 1996) p. 465.
86. U. Bleyer, V.D. Ivashchuk, V.N. Melnikov and A.I. Zhuk, *Nucl. Phys.* **B 429** 77 (1994); gr-qc/9405020.
87. V.R. Gavrilov, U. Kasper, V.N. Melnikov and M. Rainer, "Toda Chains with Type A_m Lie Algebra for Multidimensional m-component Perfect Fluid Cosmology", Preprint Math-97/ Univ. Potsdam, 1997.
88. V.R. Gavrilov V.D. Ivashchuk, and V.N. Melnikov, *Class. Quant. Grav.* **13**, 3039 (1996).
89. V.R. Gavrilov and V.N. Melnikov, *Theor. Math. Phys* to appear.
90. V.R. Gavrilov, V.N. Melnikov and R. Triay, "Exact Solutions in Multidimensional Cosmology with Shear and Bulk Viscosity", *Class. Quant. Grav.* **14**, 2203 (1997).
V.R. Gavrilov, V.N. Melnikov and M. Novello, "Exact Solutions in Multidimensional Cosmology with Bulk Viscosity", *Gravitation and Cosmology* **1**, No 2, 149 (1995).
V.R. Gavrilov, V.N. Melnikov and M. Novello, "Bulk Viscosity and Entropy Production in Multidimensional Integrable Cosmology" *Gravitation and Cosmology* **2**, No 4(8), 325 (1996).
91. M. Rainer and A. Zhuk, *Phys. Rev.*, **D 54** 6186 (1996).

92. V.D. Ivashchuk and V.N. Melnikov, "Multidimensional Gravity with Einstein Internal spaces", gr-qc/9612059; *Gravitation and Cosmology* **2** No 3(7), 211 (1996).
93. M.J. Duff, R.R. Khuri and J.X. Lu, *Phys. Rep.* **259**, 213 (1995).
94. C. Hull and P. Townsend, "Unity of Superstring Dualities", *Nucl. Phys. B* **438**, 109 (1995).
P. Horava and E. Witten, *Nucl. Phys. B* **460**, 506 (1996).
95. J.M. Schwarz, "Lectures on Superstring and M-Theory Dualities", hep-th/9607201;
96. K.S. Stelle, "Lectures on Supergravity p-Branes", hep-th/9701088.
97. M.J. Duff, "M-theory (the Theory Formerly Known as Strings)", hep-th/9608117.
98. G.W. Gibbons, G.T. Horowitz and P.K. Townsend, *Class. Quant. Grav.* **12**, 297 (1995); hep-th/9410073.
99. K.A. Bronnikov and J.C. Fabris, *Grav. and Cosmol.* **2**, No 4(8), 306 (1996).
100. A.A. Tseytlin, *Mod. Phys. Lett. A* **11**, 689 (1996).
101. G. Papadopoulos and P.K. Townsend, *Phys. Lett. B* **380**, 273 (1996).
102. A.A. Tseytlin, "Harmonic Superpositions of M-branes", hep-th/9604035; *Nucl. Phys. B* **475**, 149 (1996).
103. J.P. Gauntlett, D.A. Kastor, and J. Traschen, "Overlapping Branes in M-Theory", hep-th/9604179; *Nucl. Phys. B* **478**, 544 (1996).
104. N. Khvengia, Z. Khvengia, H. Lü, C.N. Pope, "Intersecting M-Branes and Bound States", hep-th/9605082.
105. H. Lü, C.N. Pope, and K.W. Xu, "Liouville and Toda Solitons in M-Theory", hep-th/9604058.
106. M. Cvetič and A. Tseytlin, *Nucl. Phys. B* **478**, 181 (1996).
107. V.D. Ivashchuk and V.N. Melnikov, "Intersecting p-Brane Solutions in Multidimensional Gravity and M-Theory", hep-th/9612089; *Gravitation and Cosmology* **2**, No 4, 297 (1996).
108. V.D. Ivashchuk and V.N. Melnikov, *Phys. Lett. B* **403**, 23 (1997).
109. K.A. Bronnikov and J.C. Fabris, *Gravitation and Cosmology* **2**, No 4(8), (1996).
110. E. Bergshoeff, M. de Roo, E. Eyras, B. Janssen and J.P. van der Schaar, hep-th/9612095.
111. I.Ya. Aref'eva and O.A. Rytchkov, "Incidence Matrix Description of Intersecting p-brane Solutions", hep-th/9612236.
112. R. Argurio, F. Englert and L. Hourant, "Intersection Rules for p-branes", hep-th/9701042.
113. I.Ya. Aref'eva M.G. Ivanov and O.A. Rytchkov, "Properties of Intersecting p-branes in Various Dimensions", hep-th/9702077.
114. I.Ya. Aref'eva, M.G. Ivanov and I.V. Volovich, "Non-Extremal Intersecting p-Branes in Various Dimensions", hep-th/9702079.
115. I.Ya. Aref'eva, K. Viswanathan, A.I. Volovich and I.V. Volovich, "p-Brane Solutions in Diverse Dimensions", hep-th/9701092.
116. N. Ohta, "Intersection Rules for Non-Extreme p-Branes", hep-th/9702164.
117. N. Khvengia, Z. Khvengia, H. Lü and C.N. Pope, "Toward Field Theory of

- F-Theory", hep-th/9703012.
118. V.D. Ivashchuk and V.N. Melnikov, "Sigma-Model for Generalized Composite p-branes", hep-th/9705036, to appear in *Class. and Quant. Grav.*.
 119. V.D. Ivashchuk, M. Rainer and V.N. Melnikov, "Multidimensional Sigma-Models with Composite Electric p-branes", gr-qc/9705005; to appear *Nucl. Phys.*.
 120. S.B. Chervon, *Nonlinear fields: in theory of gravitation an cosmology* (Ulyanovsk University, Ulyanovsk, 1997).
 121. K.A. Bronnikov, U. Kasper and M. Rainer, "Intersecting Electric and Magnetic p-Branes: Spherically Symmetric Solutions", gr-qc/9708058; subm. to *Phys. Rev.*.
 122. K.A. Bronnikov, M.A. Grebeniuk, V.D. Ivashchuk and V.N. Melnikov, *Grav. and Cosmol.* **3**, No 2(10), 105 (1997).
 123. V.D. Ivashchuk and V.N. Melnikov, "Multidimensional Classical and Quantum Cosmology with Intersecing p-Branes", hep-th/9708157, submitted to *J. Math. Phys.* .
 124. M.A. Grebeniuk, V.D. Ivashchuk and V.N. Melnikov, "Integrable Multidimensional Quantum Cosmology for Intersecting p-Branes", *Grav. and Cosmol.* **3**, No 3 (11), 243 (1997).
 125. K.A. Bronnikov, V.D. Ivashchuk and V.N. Melnikov, "The Reissner-Nordström Problem for Intersecting Electric and Magnetic p-Branes", gr-qc/9710054; *Grav. and Cosmol.* **3**, No 3 (11), 203 (1997).
 126. H. Lü, J. Maharana, S. Mukherji and C.N. Pope, "Cosmological Solutions, p-branes and the Wheeler De Witt Equation", hep-th/9707182.
 127. H. Lü, S. Mukherji, C.N. Pope and K.-W. Xu, "Cosmological Solutions in String Theories", hep-th/9610107.
 128. S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
 129. E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D 53**, 7020 (1996).
 130. E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D 55**, 5970 (1997).
 131. E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D 56**, (1997).
 132. E.I. Guendelman and A.B. Kaganovich, "From inflation to a zero cosmological constant phase without fine tuning", gr-qc/9709059.
 133. T. Klösch and T. Strobl, *Class. Quantum Grav.* **13** (1996), 965; Corrigendum, *ibid.* **14** (1997), 825. gr-qc/9508020
 134. T. Klösch and T. Strobl, *Class. Quantum Grav.* **13**, 2395 (1996); gr-qc/9511081.
 135. T. Klösch and T. Strobl, *Class. Quantum Grav.* **14**, 1689 (1997); hep-th/9607226.
 136. T. Klösch and T. Strobl, "A Global View of Kinks in 1+1 Gravity", gr-qc/9707053.
 137. S.S. Kokarev, submitted to *Grav. & Cosm.*.
 138. U. Günther and A. Zhuk, *Phys. Rev.* **D**, November 1997.
 139. P. G. Macedo, "Electric Neutrality and the Jordan Thiry Scalar Field", *Phys. Lett.* **A 143**, No 3, 96 (1990).
 140. P. G. Macedo, "Bounds for Jordan-Thiry Scalar Field Coupling Constant",

- Phys. Lett. A* **156**, No 1,2, 20 (1991).
141. B. Nodland and J.P. Ralston, "Indication of Anisotropy in Electromagnetic Propagation over Cosmological Distances", *Phys. Rev. Lett.* **78**, 3043 (1997).
 142. Yu.S. Vladimirov, S.I. Mamontov, "Symmetry in a 6-dimensional geometric model of gravi-electroweak interactions", *Gravitation and Cosmology* **2**, A(7), 231 (1996).
 143. V. Dzhunushaliev, gr-qc/9607007.
 144. V. Dzhunushaliev, *Izv. Vuzov, (Fizika)*, No 6, 78 (1993) (in Russian); gr-qc/9607007.
 145. S. Cotsakis, *Phys. Rev. D* **47**, 1437 (1993).
 146. S. Cotsakis, *Phys. Rev. D* **52**, 6199 (1996).
 147. S. Cotsakis and G. Flessas, *Phys. Lett. B* **319**, 69 (1993).
 148. S. Cotsakis and G. Flessas, *Phys. Rev. D* **51**, 4160 (1995).
 149. J. Miritzis and S. Cotsakis, *Phys. Lett. B* **383**, 377 (1996).
 150. S. Cotsakis, J. Miritzis and L. Querella, "Constrained variations and nonlinear gravitational lagrangians", submitted to *Class. Quant. Grav.*.
 151. S. Cotsakis and J. Miritzis, "Proof of the cosmic no-hair theorem of quadratic orthogonal Bianchi universes", submitted to *Class. Quant. Grav.*.
 152. A.M. Baranov, N.M. Bardushko, *Izv. Vuz. (Fizika)* No 1, 119 (1997) (in Russian).
 153. A.M. Baranov, *Izv. Vuz. (Fizika)* No 3, 73 (1995) (in Russian).
 154. A.M. Baranov, *Izv. Vuz. (Fizika)* No 1, 114 (1997) (in Russian).
 155. F. Burgbacher, C.Lämmerzahl and A. Macias, "Is there is stable hydrogen atom in higher dimensions?", *preprint*, University of Konstanz.
 156. C. Lämmerzahl, A. Macias, *J. Math. Phys.* **34**, 4540, (1993); Addendum: *J. Math. Phys.* **36**, 2131 (1993).
 157. J. Audretsch and C. Lämmerzahl in *Semantical Aspects of Space-Time Geometry*, ed. U. Majer and H.-J. Schmidt (BI Verlag, Mannheim, 1993).